THE THERMAL STABILITY OF A STATIONARY WAVE OF AN OPTICAL DISCHARGE

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The continuous optical discharge consists of a steady discharge sustained by a focussed continuous laser beam with a preconduction power level. Experimental investigations of the continuous optical discharge, following its discovery [1], showed that under certain conditions steady-state "burning" of the discharge does not occur: either the discharge ceases or there are periodic oscillations of the leading edge of the plasma at a frequency of the order of 10-15 Hz, which are obviously accompanied by the propagation of absorption fronts which follow one another from the ignition point in the opposite direction to the beam. These phenomena, which can be treated as the manifestation of instability of the corresponding steady-state modes of the discharge, as though it were a static discharge or a discharge wave propagating with constant velocity, are observed in the upper pressure limit at which a continuous optical discharge can exist [2].

In view of this and a number of other phenomena it is of interest to investigate the stability of the different steady-state modes of an optical discharge. The problem of the stability of any particular mode of the discharge generally arises immediately after the corresponding stationary solution is constructed. In the construction of these solutions the idea of an analogy between a burning wave – a flame front – and the discharge wave [3] and the stationary heating on the surface of a spherical volume of gas and the continuous optical discharge in a focussed beam [4] has been extremely fruitful, and has enabled the highly developed methods of the theory of combustion to be used in the physics of the discharges. It might be expected that methods of investigating the stability of flames would also be fruitful when solving the problem of the stability of different types of discharges.* In particular, the methods used in the present paper go back to [5] in which the diffusion-thermal instability of chemical-transformation fronts is investigated.

A number of qualitative effects which should affect the stability of the discharge are mentioned in [3]. In [4] a proof is given of the one-dimensional stability of a steady optical discharge of spherical shape. An attempt to analyze the stability of a continuous optical discharge to one-dimensional perturbations was also made in [6, 7], but the results obtained, in particular the conclusions regarding the existence of a limit of the stable continuous optical discharge at a fairly high temperature of the surrounding medium, are of doubtful validity due to the inconsistent use of the method of small perturbations.

Below, we investigate the problem of the linear stability of a one-dimensional wave of an optical discharge propagating towards the laser beam. A one-dimensional model of the stationary wave of the discharge is assumed [3], which is applicable when there is a small change in the light flux during the time the wave takes to travel a distance of the order of its width and assuming that the flow of gas in the channel of the light beam is one-dimensional.

We will consider the case of "intense" absorption of the electromagnetic energy flux, when the absorption length of the radiation is much less than the radius of the channel. If conditions are established in which the energy losses due to radiation and due to heat transfer through the boundary of the channel are small compared with the heat dissipation, the problem arises of the propagation of a wave of the optical discharge without loss [3]. The stationary characteristics of such a wave were obtained in [8]. We will now analyze its stability.

1. We will choose a system of coordinates attached to the propagating unperturbed wave in which the gas moves in the positive x direction (see Fig. 1). In view of the sharp Boltzmann dependence of the absorption coefficient on the temperature [3] the extent of the zone of heat dissipation in which the incident flux of electromagnetic energy of power S_0 is completely absorbed is small compared with the width of the heating zone. We

*These methods enable one to obtain an effective solution to the problem of the stability of high-frequency discharges under conditions of intense skin effect [V. I. Myshenkov, Zh. Eksp. Teor. Fiz., 73, No. 5, 1794 (1977)].

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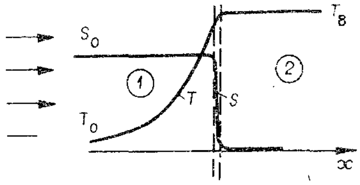

Fig. 1

will regard it as a surface which separates region 1, in which $S = S_0$, from the region of hot gas 2 with $S = 0$, which is completely screened by the absorption zone (values relating to each of these regions will henceforth be denoted by the subscripts 1 and 2, respectively).

The thermal instability will first be analyzed ignoring the influence of hydrodynamics on the discharge front and assuming constant gas density, which considerably simplifies the investigation, since in this case we do not need to consider the perturbed motion of the gas. In the following section we will show that these assumptions do not alter the results obtained regarding the stability of the discharge wave to one-dimensional perturbations.

The stationary temperature distribution in the discharge wave outside the absorption zone

$$T_1^0(x) = T_0 + (T_B - T_0)\exp(x/l) \quad (l = \lambda/\rho_0 u_0 c_p), \quad T_2^0 = T_B \tag{1.1}$$

satisfies the same heat conduction equation $lT_i^{0''} = T_i^{0'}$ ($i = 1, 2$, and the prime denotes differentiation with respect to x) and the boundary conditions $T_1^0(-\infty) = T_0$, $T_1^0(0) = T_2^0(0) = T_B$, $T_2^{0'}(\infty) = 0$. Here T_0 is the initial temperature of the gas, T_B is the temperature in the absorption zone, and λ and c_p are the thermal conductivity and specific heat at constant pressure, which are assumed to be constant, and the superscript zero here and below denotes stationary quantities. The stationary wave velocity u_0 and T_B are found from the following relations which express the law of conservation of energy and energy balance in the absorption zone:

$$S_0 = \int_{-\infty}^{+\infty} S\mu(T)\,dx, \quad \lambda T_1^{0'} = S_0, \tag{1.2}$$

where μ is the radiant-energy absorption coefficient.

Using the method of small perturbations, we will specify the perturbation of the surface of the absorption front in the form

$$\xi = \varepsilon \exp \omega t \tag{1.3}$$

(ε is a constant, ω is the increment, and t is the time) and we will seek the perturbed temperature fields satisfying the nonstationary heat-conduction equations

$$\rho_0 c_p \frac{\partial T_i}{\partial t} = \lambda \frac{\partial^2 T_i}{\partial x^2} - \rho_0 u_0 c_p \frac{\partial T_i}{\partial x} \tag{1.4}$$

in the form

$$T_i = T_i^0(x) + \delta T_i(x, t), \quad \delta T_i = f_i(x)\exp \omega t. \tag{1.5}$$

Substituting (1.5) into the linearized equations (1.4) and solving these for f_i taking into account the decay of the perturbations as $x \to -\infty$ and their boundedness as $x \to +\infty$, we obtain

$$f_1 = C_1 \exp(1 + \sqrt{1 + 4\Omega})x/2l, \; f_2 = C_2 \exp(1 - \sqrt{1 + 4\Omega})x/2l$$
$$(\Omega = \omega l/u_0, \; \mathrm{Re}\sqrt{1 + 4\Omega} > 0). \tag{1.6}$$

The perturbed solutions on the left and right of the absorption front are related to one another by the conditions of continuity of temperature T(x) and conservation of energy flux:

$$x = 0, \quad T_1^{0'}\xi + \delta T_1 = \delta T_2, \quad T_1^{0''}\xi + (\delta T_1)' = (\delta T_2)'. \tag{1.7}$$

These boundary conditions can be found by integrating the nonstationary heat-conduction equation

$$\rho_0 c_p \frac{\partial T}{\partial t} + \rho_0 u_0 c_p \frac{\partial T}{\partial x} = \lambda \frac{\partial^2 T}{\partial x^2} + \mu S \tag{1.8}$$

with the added term μS representing the heat dissipation due to absorption of the electromagnetic energy flux in the absorption zone, allowing its thickness to approach zero, and linearizing the relations obtained taking into account the fact that ξ and δT are quantities of the first order of smallness. In deriving the second boundary condition in (1.7) allowance must be made for the fact that the source term in (1.8) is equal to $-S'$ in view of the equation of light-flux absorption $S' = -\mu S$.

In the linear approximation the change in the velocity of propagation of the stationary discharge wave, which can be assumed to depend only on the temperature in the absorption zone, is represented by the coefficient

$$z = (T_B - T_0) d \ln u_0 / dT_B, \tag{1.9}$$

the explicit expression for which is determined by the specific form of the $u_0(T_B)$ relationship [for $u_0 \sim \exp(-I/2kT_B)$ we have $z = I(T_B - T_0)/2kT_B^2$]. Then the linearized condition for total absorption of the light flux in the wave front (taking into account the smallness of ξ) can be written in the form

$$\frac{\partial \xi}{\partial t} = -z \frac{u_0}{T_B - T_0} \delta T_2(0). \tag{1.10}$$

Substituting (1.3) and the solutions (1.5) and (1.6) into Eqs. (1.7) and (1.10) we obtain a system of three linear equations for ε, C_1 and C_2, which has a solution provided

$$2\Omega\sqrt{1+4\Omega} + z(\sqrt{1+4\Omega} - 1) = 0. \tag{1.11}$$

It can be shown that this equation does not have unstable roots with $\mathrm{Re}\,\Omega > 0$, i.e., the optical-discharge wave is stable for all z. The root $\Omega = 0$ of Eq. (1.11) represents perturbations which occur when the initial stationary temperature perturbation is displaced in the x direction without changing its shape. The existence of such perturbations follows from the invariance of the heat-conduction equation employed with respect to a displacement along the x axis. In [9], which is devoted to an analysis of the one-dimensional stability of a laminar flame, it was shown that the presence of such a "translational" perturbation does not indicate that the stationary solution is unstable.

The above investigation is broadly similar to the analysis of the stability of the front of an exothermal chemical reaction propagating in a condensed medium [10]. The difference, due to the specific features of the system considered, lies solely in the second boundary condition in (1.7).

2. We will carry out a more accurate analysis of the stability taking into account the dependence of the gas density on temperature. The gas is assumed to be dynamically incompressible, since the rate of propagation of the wave is much less than the velocity of sound (the Mach number is much less than unity). The pressure drop in the wave front is small (of the order of the square of the Mach number), and the pressure can be assumed to be constant. The gas density is then found from the known temperature distribution (1.1) using the equation of state

$$\rho T = \mathrm{const} = \rho_0 T_0, \quad \text{i.e.} \quad \rho_1^0 = \rho_0 T_0 / T_1^0, \quad \rho_2^0 = \rho_0 T_0 / T_B \tag{2.1}$$

(the superscript zero denotes quantities at $x = -\infty$).

The stationary velocity of the gas is found from the equation of continuity

$$\rho^0 u^0 = \mathrm{const} = \rho_0 u_0, \quad u_1^0 = \rho_0 u_0 / \rho_1^0, \quad u_2^0 = \rho_0 u_0 / \rho_2^0 = u_0 T_B / T_0. \tag{2.2}$$

When analyzing the stability one must bear in mind the fact that the temperature perturbation, according to the equation of state (2.1), leads to a perturbation of the density $\delta\rho_i = -(\rho_0 T_0 / T_i^{0^2}) \delta T_i$ $(\delta\rho_i = \rho_i - \rho_i^0)$. Hence, the linearized heat-conduction and continuity equations take the form

$$\begin{gathered}\rho_i^0 c_p \frac{\partial \delta T_i}{\partial t} + \rho_0 u_0 c_p \frac{\partial \delta T_i}{\partial x} + c_p \frac{dT_i^0}{dx} \delta(\rho u)_i = \lambda \frac{\partial^2 \delta T_i}{\partial x^2}, \\ -\frac{\rho_0 T_0}{T_i^{0^2}} \frac{\partial \delta T_i}{\partial t} + \frac{\partial}{\partial x} \delta(\rho u)_i = 0 \quad (\delta(\rho u)_i = \rho u - \rho_0 u_0).\end{gathered} \tag{2.3}$$

Substituting (1.5) and $\delta(\rho u)_i = \rho_0 u_0 \varphi_i(x) \exp \omega t$ we obtain

$$\frac{T_0}{T_i^0} \Omega f_i + l f_i' + l T_i^{0\prime} \varphi_i = l^2 f_i''; \tag{2.4}$$

$$-\frac{T_0}{T_i^{0^2}} \Omega f_i + l \varphi_i' = 0. \tag{2.5}$$

We will obtain the solution of this system of equations in the region $x < \xi$ (i = 1). Multiplying Eq. (2.5) by T_1^0 and adding it to (2.4) we obtain the integral of the system

$$\varphi_1 = (lf_1' - f_1)/T_1^0 \tag{2.6}$$

(the constant of integration is zero since $f_1 = 0$, $f_1' = 0$, $\varphi_1 = 0$ when $x = -\infty$).

Substituting φ_1 from Eq. (2.6) into Eq. (2.5) we obtain a second-order linear differential equation in f_1 with variable coefficients

$$f_1'' - a_1(x) f_1' + a_2(x) f_1 = 0, \quad a_1 = \frac{1}{l} + \frac{T_1^{0\prime}}{T_1^0}, \quad a_2 = \frac{1}{l^2}\frac{lT_1^{0\prime} - \Omega T_0}{T_1^0}, \tag{2.7}$$

which must satisfy the boundary condition $f_1(-\infty) = 0$.

It is not possible to integrate Eq. (2.7). However, we can use the fact that to analyze the stability it is sufficient to construct a solution of Eq. (2.7) with the condition $f_1(-\infty) = 0$ in the neighborhood of the point $x = 0$ with an accuracy to $O(x)$.

We will proceed as follows. We obtain the general solution (2.7) assuming a_1 and a_2 to be constant:

$$\begin{gathered} f_1 = C_1 y_1 + \tilde{C}_1 y_2, \quad y_i = \exp \beta_i(x), \\ \beta_{1,2} = \frac{1}{2} a_1 \pm \sqrt{\frac{1}{4} a_1^2 - a_2}, \quad \operatorname{Re} \sqrt{\frac{1}{4} a_1^2 - a_2} > 0 \end{gathered} \tag{2.8}$$

(C_1 and $\tilde{C}_1$ are constants).

If we now take into account the fact that the coefficients a_1 and a_2 depend on x, Eq. (2.8) becomes an approximate solution which coincides with the true solution as $x \to -\infty$, since Eq. (2.8) is the solution of Eq. (2.7) with the coefficients $a_1(-\infty)$ and $a_2(-\infty)$. In view of the boundary condition $f_1(-\infty) = 0$ we must put $\tilde{C}_1 = 0$.

In the region $x \to -\infty$ the function $f_1 = C_1 y_1$ with $\beta_1(x) = \beta_1(0)$ describes the solution of the initial equation (2.7) very accurately [the error which occurs on substituting the approximate solution y_1 into Eq. (2.7) does not exceed a quantity of the order $\beta_1'(0) \approx \sqrt{T_0/T_B} \ll 1$ of the value of the main term] and, in the required approximation, has the form

$$f_1 = C_1(1 + \beta_1(0)x), \quad \beta_1(0) = \frac{1}{l}\left[1 - \frac{T_0}{2T_B} + \frac{T_0}{2T_B}\sqrt{1 + 4\frac{T_B}{T_0}\Omega}\right]. \tag{2.9}$$

At other points the approximate solution (2.8) may differ considerably from the true solution, but for our purposes it is only important that it should be valid for small and large values of $|x|$.

In the region $x > \xi$ the solution for f_2 can immediately be found from Eq. (2.4) since $T_2^{0\prime} = 0$. Taking into account the fact that the perturbations are bounded at $x = +\infty$ it has the form

$$f_2 = C_2 \exp\left(1 - \sqrt{1 + 4\frac{T_0}{T_B}\Omega}\right) x/2l. \tag{2.10}$$

Substituting the solutions (2.9), (2.10) and (1.3) into the boundary conditions on the discharge front (1.7) and, taking into account the temperature dependence of the density, the altered condition for total absorption of the light flux in the wave front

$$\frac{\partial \xi}{\partial t} = -\frac{u_0}{T_0}\left(1 + z\frac{T_B}{T_B - T_0}\right)\delta T_2(0),$$

we obtain the following condition for the system of three linear equations in C_1, C_2 and ε to have a solution:

$$\Omega\left[\sqrt{1 + 4\frac{T_B}{T_0}\Omega} + 1 + \frac{T_0}{T_B}\left(\sqrt{1 + 4\frac{T_B}{T_0}\Omega} - 1\right)\right] + \left(\frac{T_B - T_0}{T_B} + z\right)\left(\sqrt{1 + 4\frac{T_B}{T_0}\Omega} - 1\right) = 0. \tag{2.11}$$

This equation does not have unstable roots with $\operatorname{Re}\Omega > 0$ (as mentioned above the root $\Omega = 0$ is not unstable), i.e., the conclusion in Sec. 1 regarding the stability of the discharge wave obtained when analyzing the perturbations of only the temperature field is confirmed. The stability is not disturbed if allowance is made for the fact that the gas expands due to it being heated by the heat dissipated in the absorption zone.

From Eqs. (2.6) and (2.5) for i = 2 we can obtain the perturbations of the mass flow of gas in regions 1 and 2, which are connected to one another by the condition of continuity of the mass flow through the wave front. The corresponding equation contains the constants C_1 and C_2 and one new constant equal to the perturbation of the gas velocity behind the wave front when $x = \infty$. It is not used to obtain the characteristic equation (2.11) and is therefore not given here.

3. One-dimensional perturbations of the discharge-wave front are the most dangerous from the point of view of the occurrence of instability. In fact, when the surface of the absorption front is distorted, corresponding to spatial (non-one-dimensional) perturbations, the parts of the front that are convex in the direction of propagation of the wave dissipate heat more than the plane parts of the front. Hence, the velocity of propagation of the discharge front, which depend strongly on the temperature, are less stationary on the convex parts. On the concave parts the gas temperature on the other hand, is greater than on the convex parts, since the gas is heated additionally from the neighboring convex parts. The velocity of propagation of the concave parts is correspondingly greater than the stationary part. On the whole, as a result of the increase in the velocity on the concave parts and the reduction in the velocity on the convex parts the discharge front becomes equalized. Hence, when the front is distorted, additional stabilization of the discharge wave occurs. A mathematical investigation of the stability of the discharge front with respect to spatial perturbations can be carried out within the framework of the model investigated in Sec. 1. In this case perturbation of the surface of the front is specified in the following form (three-dimensional perturbations can be reduced to two-dimensional)

$$\xi = \varepsilon \exp(\omega t + iky),$$

where y is the coordinate directed along the front, and k is the wave number of the perturbation. On the left side of the heat-conduction equation (1.4) one must add the term $\lambda \partial^2 T/\partial y^2$ and the solution of this equation must be sought in the form

$$\delta T_i = f_i(x) \exp(\omega t + iky).$$

Then, in the solutions (1.6), instead of Ω one must have the expression $\Omega + K^2$ ($K = lk$). The boundary conditions remain unchanged. As a result, in Eq. (1.11) instead of Ω we have the expression $\Omega + K^2$ and the decrement of the principal most slowly decaying solution will be

$$\Omega = -K^2,$$

which confirms that there is a certain stability reserve due to the two-dimensional nature of the perturbations, in accordance with the physical mechanism suggested above.

If the wave propagates under conditions when focussing of the laser beam is considerable, on the convex parts of the perturbed surface of the front the power of the light flux and consequently the heat dissipation will be less than on the concave parts, which should lead to additional stabilization of the wave front and to more more rapid equalization.

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HEAT EXCHANGE OF A CYLINDER WITH LOW-FREQUENCY OSCILLATIONS

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It is well known that the presence of a sonic field intensifies heat-mass exchange processes [1-3], and that this intensification is due to the presence of stationary secondary flows formed near the solid surface [1]. However, existing theoretical treatments of this question are limited to the case of high-frequency oscillations, while the situation in which the thickness of the Stokes layer is comparable to or larger than the size of the body is no less important. For example, such a situation is realized in heating devices operating in a high-frequency instability regime and using atomized liquid or solid fuel. These problems are of importance in thermoanemometry. In the present study the example of a circular cylinder will be used to study the effect of low-frequency oscillations on local and integral characteristics of the heat exchange process. By low frequency, we refer to the region where the Stokes layer thickness $[\delta_{ac} \sim (\nu/\omega)^{0.5}]$ is comparable to or larger than the cylinder size.

Let a circular cylinder of radius a and infinite length be located within an infinite viscous liquid, which at an infinite distance from the cylinder undergoes oscillations following a harmonic law with cyclical frequency ω. The temperatures of the cylinder surface $\tilde{T}_W$ and the surrounding medium $\tilde{T}_\infty$ are considered constant, and the temperature difference $(\tilde{T}_W - \tilde{T}_\infty)$ is assumed so small that changes in the physical properties of the liquid and natural convection may be neglected. Also neglecting dissipative effects, we write the energy equation in the form [3]:

$$\frac{\partial T}{\partial \tau} + \frac{\varepsilon}{1+r}\frac{\partial(\psi, T)}{\partial(r, \theta)} = \frac{H^2}{\Pr}\nabla^2 T \tag{1}$$

with boundary conditions

$$T = 1 \text{ for } r = 0, \ T = 0 \text{ for } r \to \infty. \tag{2}$$

The dimensionless quantities in Eqs. (1), (2) are defined as follows:

$$r = (\tilde{r} - a)/a, \ \psi = \tilde{\psi}/Ba, \ \tau = \tilde{\tau}\omega, \ T = (\tilde{T} - \tilde{T}_\infty)/(\tilde{T}_w - \tilde{T}_\infty),$$

where $\varepsilon = S/a$; $H = \delta_{ac}/a$; $\delta_{ac} = \sqrt{\nu/\omega}$; S is the amplitude of the acoustical displacement of the medium; $B = S\omega$ is the amplitude of the velocity pulsations. The tilda superscript denotes quantities having dimensions.

We will consider the case in which $\varepsilon \ll 1$ (a similar assumption was used in solving the hydrodynamic portion of the problem [4]). Then, using the perturbation method, we write the solution of Eq. (1) in the form of a series

$$T = T_0 + \varepsilon T_1 + O(\varepsilon^2) \tag{3}$$

and similarly represent the velocity field

$$\psi = \psi_0 + \varepsilon\psi_1 + O(\varepsilon^2). \tag{4}$$

We recall that according to [4], ψ_0 is a periodic function of time with frequency ω and contains no time-independent component, while ψ_1 consists of two components, a stationary ψ_1^{st} and a periodic ψ_1^{p}, which varies with a cyclical frequency 2ω.

Since we are interested in the effect of low-frequency oscillations on the heat exchange of the circular cylinder, we will assume further that $H = O(1)$.

We will consider the case where $\Pr = O(1)$. Substituting Eqs. (3), (4) in Eq. (1) and collecting terms with identical powers of ε, we obtain the following equations:

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